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New mathematical insights into Lagrangian turbulence

A sneeze. Ocean currents. Smoke. What do these have in common? They're instances of turbulence: unpredictable, chaotic, uneven fluid flows of fluctuating velocity and pressure. Though ubiquitous in nature, these flows remain somewhat of a mystery, theoretically and computationally.

"Most flows that we encounter in nature are turbulent — it does not matter whether it is the flow outside the airplane that makes us fasten our seatbelts, or the flow in a small stream," said UC Santa Barbara mathematics professor Björn Birnir. "Turbulence is difficult to understand because the mathematical models that describe it are nonlinear, stochastic and the solutions are unstable. This made it necessary to develop new theories to truly understand the nature of turbulence."

Fortunately, Birnir and Luiza Angheluta of the University of Oslo are getting us closer to being able to characterize turbulence, with an approach that captures some of the myriad complex phenomena that occur over the evolution of a turbulent flow. Their research is published in the journal Physical Review Research.

'The most important unsolved problem'

Described in 1964 by famed physicist Richard Feynmann as "the most important unsolved problem of classical physics," turbulence has accumulated its fair share of laws and theories, as researchers over two centuries contributed valuable insights and approaches to the study of this highly complex phenomenon. However, because of its nonlinearity, general unpredictability and also its multi-scale nature, generating the math that holds true for everything from the tiniest fluctuation to the entire flow with all its interacting vortices and eddies has been one of the primary challenges of the field.

This is particularly true of the turbulent flow called Lagrangian turbulence, where an observer follows the flow (as in an airplane). It starts by an initial ballistic flow (all particles stuck together and flowing in the same direction), goes through large Lagrangian vortices and later Eulerian turbulence (a homogeneous flow with smaller but more complex vortices).

“There has been a lot of speculation,” said Birnir, who directs the Center for Complex and Nonlinear Science at UCSB. “The ballistic region has a certain scaling. The

Lagrangian region has another scaling, and then it looked like it was going into this region where there was Eulerian scaling.” Each scaling regime contains the math that best describes the forces and unique phenomena in only that particular evolution of the turbulent flow. “So you are basically seeing three types of scalings but there was no theory behind it and there was in fact no proof of it.” Instead of becoming clearer, the study of turbulence had become more confusing, he added.

While the ballistic and Eulerian flows have fairly well-established scaling laws, the region between them was relatively less understood.

“Different scaling regions, in time, is one of the main characteristics of Lagrangian turbulence,” Birnir explained. Another unique characteristic is the Lagrangian framework’s approach, which is to follow the turbulence from the point of view of a particle — a “tracer” — within the flow, as opposed to from a stationary point outside the flow, as is the case with Eulerian turbulence, where the flow is more homogenous.

"Big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity."

— Lewis Fry Richardson, considered to be the father of modern Lagrangian turbulence theory

Birnir and Angheluta investigated the statistical properties of a fully turbulent Lagrangian velocity field using a modeling framework called stochastic closure theory, which captures randomness as part of the system. They also used a set of relations called the Green-Kubo-Obukhov relations to characterize the effects of various forces on and conditions of the flow such as diffusion and viscosity, as well as the chaotic dynamics of the entire system.

The result is a mathematical model that demonstrates the presence of a Lagrangian scaling regime in the "passover region" between ballistic flows and Eulerian turbulence, while also connecting the three scaling regimes as the turbulent flow evolves from its initial conditions through the ballistic region, as it superdiffuses into the chaotic, multi-scale fluctuations and flows of the Lagrangian region and transitions to the more homogenous Eulerian region. Additionally, the researchers identify a fourth region "free eddies" — free-floating, rapidly swirling vortices that are disconnected from the earlier turbulence. Their results, according to an introduction of their work in Physical Review Research, "show excellent agreement with Direct Navier-Stokes simulations."

This enhanced statistical understanding of Lagrangian turbulence will be useful for tackling real-world puzzles of turbulence, such as tracking ocean currents, and predicting weather patterns and how pollutants and airborne pathogens spread.

"This gives us a little more foundation for calculating things like the spread of COVID and other aerosols," said Birnir, who plans to write a biomedical paper to provide information on how to use this model to better calculate the infectiousness of diseases carried by airborne pathogens.

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