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Sharpening Occam's Razor

In science, the explanation with the fewest assumptions is most likely to be true. Called "Occam's Razor," this principle has guided theory and experiment for centuries. But how do you compare between abstract concepts?

In a new paper, philosophers from UC Santa Barbara and UC Irvine discuss how to weigh the complexity of scientific theories by comparing their underlying mathematics. They aim to characterize the amount of structure a theory has using symmetry — or the aspects of an object that remain the same when other changes are made.

After much discussion, the authors ultimately doubt that symmetry will provide the framework they need. However, they do uncover why it's such an excellent guide for understanding structure. Their paper appears in the journal Synthese.

"Scientific theories don't often wear their interpretation on their sleeves, so it can be hard to say exactly what they're telling you about the world," said lead author Thomas Barrett, an associate professor in UC Santa Barbara's philosophy department. "Especially modern theories. They just get more mathematical by the century." Understanding the amount of structure in different theories can help us make sense of what they're saying, and even give us reasons to prefer one over another.

Structure can also help us recognize when two ideas are really the same theory, just in different clothes. For instance, in the early 20th century, Werner Heisenberg and

Erwin Schrödinger formulated two separate theories of quantum mechanics. “And they hated each other’s theories,” Barrett said. Schrödinger argued that his colleague’s theory “lacked visualizability.” Meanwhile, Heisenberg found Schrödinger’s theory “repulsive” and claimed that “what Schrödinger writes about visualizability [...] is crap.”

But while the two concepts appeared radically different, they actually made the same predictions. About a decade later, their colleague John von Neumann demonstrated that the formulations were mathematically equivalent.

Apples and oranges

A common way to examine a mathematical object is to look at its symmetries. The idea is that more symmetric objects have simpler structures. For instance, compare a circle — which has infinitely many rotational and reflective symmetries — to an arrow, which has only one. In this sense, the circle is simpler than the arrow, and requires less mathematics to describe.

The authors extend this rubric to more abstract mathematics using automorphisms. These functions compare various parts of an object that are, in some sense, “the same” as each other. Automorphisms give us a heuristic for measuring the structure of different theories: More complex theories have fewer automorphisms.

In 2012, two philosophers proposed a way to compare the structural complexity of different theories. A mathematical object X has at least as much structure as another, Y, if and only if the automorphisms of X are a subset of those of Y. Consider the circle again. Now compare it to a circle that is colored half red. The shaded circle now has only some of the symmetries it used to, on account of the extra structure that was added to the system.

This was a good try, but it relied too much on the objects having the same type of symmetries. This works well for shapes, but falls apart for more complicated mathematics.

Isaac Wilhelm, at the National University of Singapore, attempted to fix this sensitivity. We should be able to compare different types of symmetry groups as long as we can find a correspondence between them that preserves each one’s

internal framework. For example, labeling a blueprint establishes a correspondence between a picture and a building that preserves the building's internal layout.

The change allows us to compare the structures of very different mathematical theories, but it also spits out incorrect answers. "Unfortunately, Wilhelm went a step too far there," Barrett said. "Not just any correspondence will do."

A challenging endeavor

In their recent paper, Barrett and his co-authors, JB Manchak and James Weatherall, tried to salvage their colleague's progress by restricting the type of symmetries, or automorphisms, they would consider. Perhaps only a correspondence that arises from the underlying objects (e.g. the circle and the arrow), not their symmetry groups, is kosher.

Unfortunately, this attempt fell short as well. In fact, it seems that using symmetries to compare mathematical structure may be doomed by principle. Consider an asymmetric shape. An ink blot, perhaps. Well, there's more than one ink stain in the world, all of which are completely asymmetric and completely different from one another. But, they all have the same symmetry group — namely, none — so all these systems classify the ink blots as having the same complexity even if some are far messier than others.

This ink blot example reveals that we can't tell everything about an object's structural complexity just by looking at its symmetries. As Barrett explained, the number of symmetries an object admits bottoms out at zero. But there isn't a corresponding ceiling to the amount of complexity an object can have. This mismatch creates the illusion of an upper limit for structural complexity.

And therein the authors expose the true issue. The concept of symmetry is powerful for describing structure. However, it doesn't capture enough information about a mathematical object — and the scientific theory it represents — to allow for a thorough comparison of complexity. The search for a system that can do this will continue to keep scholars busy.

A glimmer of hope

While symmetry might not provide the solution the authors hoped for, they uncover a key insight: Symmetries touch on the concepts that an object naturally and organically comes equipped with. In this way, they can be used to compare the structures of different theories and systems. “This idea gives you an intuitive explanation for why symmetries are a good guide to structure,” Barrett said. The authors write that this idea is worth keeping, even if philosophers have to abandon using automorphisms to compare structure.

Fortunately, automorphisms aren’t the only kind of symmetry in mathematics. For instance, instead of looking only at global symmetries, we can look at symmetries of local regions and compare these as well. Barrett is currently investigating where this will lead and working to describe what it means to define one structure in terms of another.

Although clarity still eludes us, this paper gives philosophers a goal. We don’t know how far along we are in this challenging climb to the summit of understanding. The route ahead is shrouded in mist, and there may not even be a summit to reach. But symmetry provides a hold to anchor our ropes as we continue climbing.

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edge of the Pacific Ocean.